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There is evidently but one solution; for if the other point of intersection of the circles be taken instead of C, the resulting quadrilateral will be the same in size but drawn on the opposite side of AD since the intersections are symmetrical with regard to AD.

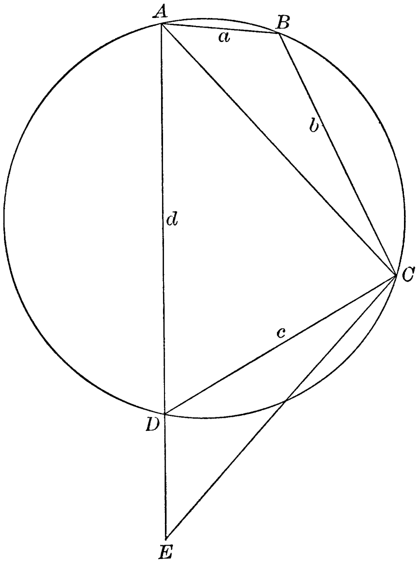

Fig. 3.

Note.—In this construction it is assumed that the order of the sides a, b, c, d, is given. If this order is not given it may be shown that three cyclic quadrilaterals can be drawn having these sides.

MECHANICS.

265. Proposed by A. H. HOLMES, Brunsw'ck, Maine.

A gun is mounted in a fort at height h above the sea, and a similar gun is mounted on a ship. Show that there is a region of area $4\pi rh$ within which the ship is within range of the fort while the fort is out of range of the ship, r being the maximum range of either gun on a horizontal plane through it.

SOLUTION BY A. M. HARDING, University of Arkansas.

If a projectile is fired at an angle α its range R on a level plane at a distance h below the point of projection is given by the equation

$$2v^2(h + R\tan\alpha) = gR^2\sec^2\alpha.$$

That is

$$R = \frac{v}{g}\cos\alpha\,[v\sin\alpha + \sqrt{(v^2 + 2gh) - v^2\cos^2\alpha}]\cdot$$

From the value of $dR/d\alpha$, set equal to zero, we have as the condition for a maximum range,

$$\sin^2\alpha = \frac{v^2}{2(v^2+gh)} \qquad \text{and} \qquad \cos^2\alpha = \frac{v^2+2gh}{2(v^2+gh)}\cdot$$

Hence $R = \sqrt{r^2 + 2hr}$, where r is the maximum horizontal range. This is the maximum range for a gun in the fort. Likewise the maximum range for a gun on the ship is

$$R_1 = \sqrt{r^2 - 2hr}.$$

Therefore the area required is

$$\pi R^2 - \pi R_1^2 = 4\pi hr.$$

Also solved by MRS. H. E. TREFETHEN and H. C. FEEMSTER.

NUMBER THEORY.

185. Proposed by R. D. CARMICHAEL, Bloomington, Ind.

Obtain the complete solution of the equation $\phi(p^\alpha) = \phi(q^\beta)$ where ϕ denotes Euler's ϕ-function, p and q are unknown primes, and α and β are unknown integers.

SOLUTION BY L. C. MATHEWSON.

From the theory of numbers[1] we have

$$\begin{aligned} \phi(p^\alpha) &= p^\alpha\left(1 - \frac{1}{p}\right) = p^{\alpha-1}(p-1); \\ \phi(q^\beta) &= q^\beta\left(1 - \frac{1}{q}\right) = q^{\beta-1}(q-1). \end{aligned} \tag{1}$$

Obviously the given equation is always satisfied by $p = q$, if $\alpha = \beta$. We, accordingly, consider the case where the primes are not equal, or $p \neq q$.

From (1) we have

$$p^{\alpha-1}(p-1) = q^{\beta-1}(q-1). \tag{2}$$

Since $p^{\alpha-1}$ and $q^{\beta-1}$ can have no common factors, we see that (2) gives

$$\begin{aligned} p^{\alpha-1} &= q - 1 \\ q^{\beta-1} &= p - 1 \end{aligned}. \tag{3}$$

It is sufficient to consider only the first of these equations, since the second is of the same form and leads to no new general results or solutions. Now $\alpha = 1$ or $\alpha > 1$. From (3) written in the form $p^{\alpha-1} + 1 = q$, we see that if $\alpha = 1$, $q = 2$, the even prime: if $\alpha > 1$, p cannot be an odd prime (for this would make q composite), hence p can be only 2. Since we are now excluding $p = q$, and because from (3) we have seen that the results will be interchangeable, and since we now see that at least one of the primes here must be the even prime, we consider p odd and q even. We have then to discuss $\phi(p^\alpha) = \phi(q^\beta)$, where p is an odd prime, $\alpha = 1$, $q = 2$; i. e., $\phi(p) = \phi(2^\beta)$.

We have simply $p - 1 = 2^{\beta-1}$, or $p = 2^{\beta-1} + 1$. Here p is an odd prime of the form $2^m + 1$. It is easily seen that m cannot be an odd prime nor an integral multiple of an odd prime (for in these cases since the binomial form itself is the

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[1] See Bachmann's Neuere Zahlentheorie, p. 24, or any elementary text on the theory of numbers.